Formation of General Position by Asynchronous Mobile Robots

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ABSTRACT

The traditional distributed model of autonomous, homogeneous, mobile point robots usually assumes that the robots do not create any visual obstruction for the other robots, i.e., the robots are see through. In this paper, we consider a slightly more realistic model, by incorporating the notion of *obstructed visibility* (i.e., robots are not see through) for other robots. Under the new model of visibility, a robot may not have the full view of its surroundings. Many of the existing algorithms demand that each robot should have the complete knowledge of the positions of other robots. Since, vision is the only mean of their communication, it is required that the robots are in *general position* (i.e., no three robots are collinear). We consider *asynchronous* robots. They also do not have common *chirality* (or any agreement on a global coordinate system). In this paper, we present a distributed algorithm for obtaining a general position for the robots in finite time from any arbitrary configuration. The algorithm also assures collision free motion for each robot. This algorithm may also be used as a preprocessing module for many other subsequent tasks performed by the robots.

Keywords

Asynchronous, oblivious, obstructed visibility, general position.

1. INTRODUCTION

The study of a set of autonomous mobile robots, popularly known as swarm robots or multi robot system, is an emerging research topic in last few decades. Swarm of robots is a set of autonomous robots that have to organize themselves in order to execute a specific task in collaborative manner. Various problems in several directions, have been studied in the framework of swarm robots, among the others distributed computing is an important area with this swarm robots. This paper explores that direction.

1.1 Framework

The traditional distributed model [12] for multi robot system, represents the mobile entities by distinct points located in the Euclidean plane. The robots are anonymous, indistinguishable, having no direct means of communication. They have no common agreement in directions, orientation and unit distance. Each robot has sensing capability, by *vision*, which enables it to determine the position (within its own coordinate system) of the other robots. The robots operate in rounds by executing *Look-Compute-Move* cycles. All robots may or may not be active at all rounds. In a round, when becoming active, a robot gets a snapshot of its surroundings (Look) by its sensing capability. This snapshot is used to compute a destination point (Compute) for this robot. Finally, it moves towards this destination (Move). The robot either directly reaches destination or moves atleast a small distance towards the destination. The choice of active robot in each round is decided by an adversary. However, it is guaranteed that each robot will become active in finite time. All robots execute the same algorithm. The robots are oblivious, i.e., at the beginning of each cycle, they forget their past observations and computations [10]. Depending on the activation schedule and the duration of the cycles, three models are defined. In the *fully-synchronous* model, all robots are activated simultaneously. As a result, all robots acts on same data. The *semi-synchronous* model is like the fully synchronous, except that the set of robots to be activated is chosen at random. As a result, the active robots act on same data. No assumption, is made on timing of activation and duration of the cycles for *asynchronous* model. However, the time and durations are considered to be finite.

Vision and mobility enable the robots to communicate and coordinate their actions by sensing their relative positions. Otherwise, the robots are silent and have no explicit message passing. These restrictions enable the robots to be deployed in extremely harsh environments where communication is not possible, i.e an underwater deployment or a military scenario where wired or wireless communications are impossible or can be obstructed or erroneous.

1.2 Earlier works

Majority of the investigations[9, 12] on mobile robots assume that their visibility is unobstructed or full, i.e., if two robots A and B are located at a and b, they can see each other though other robots lie in the line segment $\overline{ab}$ at that time. Very few observations on obstructed visibility (where

A and B are not mutually visible if there exist other robots on the line segment $\overline{ab}$) have been made in different models; such as, (i) the robots in the one dimensional space [5]; (ii) the robots with visible lights [7, 8] and (iii) the unit disc robot called *fat robots* [1, 6].

The first model studied the uniform spreading of robots on a line [5]. In the second model, each agent is provided with a local externally visible *light*, which is used as colors [7, 8, 9, 11, 12, 13, 2]. The robots implicitly communicate with each other using these colors as indicators of their states. In the third model, the robots are not points but unit discs [4, 6, 1]) and collisions among robots are allowed.

Obstructed visibility have been addressed recently in [2] and [3]. In [2] the authors have proposed algorithm for robots in light model. Here, the robots starting from any arbitrary configuration form a circle which is itself an unobstructed configuration. The presence of a constant number of visible light(color) bits in each robot, implicitly help the robots in communication and storing the past configuration. In [2], the robots obtain a obstruction free configuration by getting as close as possible. Here, the robots do not have light bits. However, the algorithm is for semi-synchronous robots.

1.3 Our Contribution

In this paper, we propose algorithm to remove obstructed visibility by making of general configuration by the robots. The robots start from arbitrary distinct positions in the plane and reach a configuration when they all see each other. The robots are asynchronous, oblivious, having no agreement in coordinate systems. The obstructed visibility model is no doubt improves the traditional model of multi robot system by incorporating real-life like characteristic. The problem is also a preliminary step for any subsequent tasks which require complete visibility.

The organization of the paper is as follows: Section 2, defines the assumptions of the robot model used in this paper and presents the definitions and notations used in the algorithm. Section 3 presents an algorithm for obtaining general position by asynchronous robots. We also furnish the correctness of our algorithm in this section. Finally in section 4 we conclude by providing the future directions of this work.

2. MODEL AND DEFINITIONS

Let $\mathcal{R} = \{r_1 \ldots, r_n\}$ be a set of n homogeneous robots represented by points. Each robot can sense (see) 360° around itself up to an unlimited radius. However, they obstruct the visibility of other robots. The robots execute *look-compute-move* cycle in *asynchronous* manner. They are *oblivious* and have *no direct communication* power. The movement of the robots are *non-rigid*, i.e., a robot may stop before reaching its destination. However, a robot moves at-least a minimum distance $\delta > 0$ towards its destination. This assumption assures that a robot will reach its destination in finite time. Initially the robots are positioned in distinct locations and are stationary. Now we present some notations and conventions which will be used throughout the paper.

- **Position of a robot:** $r_i \in \mathcal{R}$ represents a location of a robot in $\mathcal{R}$ at some time, i.e., r_i is a position occupied by a robot in $\mathcal{R}$ at certain time. To denote a robot in $\mathcal{R}$ we refer by its position r_i.

- **Measurement of angles:** By *an angle between two*

line segments, if otherwise not stated, we mean the angle made by them which is less than equal to π.

- $\mathbf{\mathcal{V}(r_i)}$: For any robot r_i, we define the vision of r_i, $\mathcal{V}(r_i)$, as the set of robots visible to r_i (excluding r_i itself). The robots in $\mathcal{V}(r_i)$ can also be in motion due to asynchronous scheduling.

  If we sort the robots in $\mathcal{V}(r_i)$ by angle at r_i, w.r.t. r_i and connect them in that order, we get a star-shaped polygon, denoted by $STR(r_i)$. Note that $r_j \in \mathcal{V}(r_i)$ if and only if $r_i \in \mathcal{V}(r_j)$ (Figure 1).
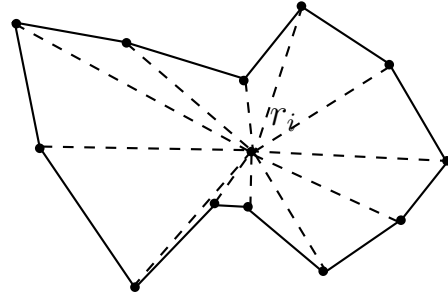

Figure 1: An example of STR(r$_i$)

- $\mathbf{CR(r_i)}$: This is the set of line segments joining r_i to all its neighbors or all robots in $\mathcal{V}(r_i)$. $CR(r_i) = \{\overline{r_i r_j} : r_j \in \mathcal{V}(r_i)\}$ (Figure 2).
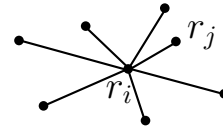

Figure 2: An example of CR(r$_i$)

- $\mathbf{\mathcal{L}_{r_i r_j}}$: Straight line through r_i and r_j : $r_j \in \mathcal{V}(r_i)$ (Figure 3)

- $\mathbf{COL(r_i)}$: $COL(r_i)$ denotes the set of robots for which r_i creates visual obstructions.

- $\mathbf{DISP(r_i r_j)}$: When a robot r_i moves to new position $\hat{r}_i$, we call $\angle r_i r_j \hat{r}_i$ as the angle of displacement of r_i w.r.t. r_j and denote it by $DISP(r_i r_j)$ (Figure 3).
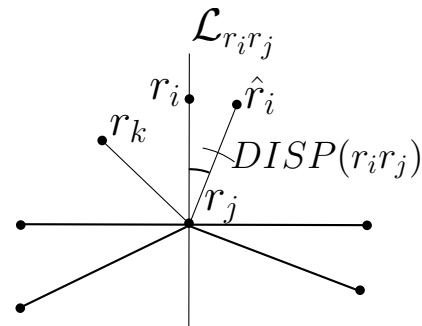

Figure 3: Examples of $\mathcal{L}_{r_i r_j}$, DISP(r$_i$r$_j$) = $\angle$r$_i$r$_j$r̂$_i$, COL(r$_i$) = {r$_l$, r$_m$}
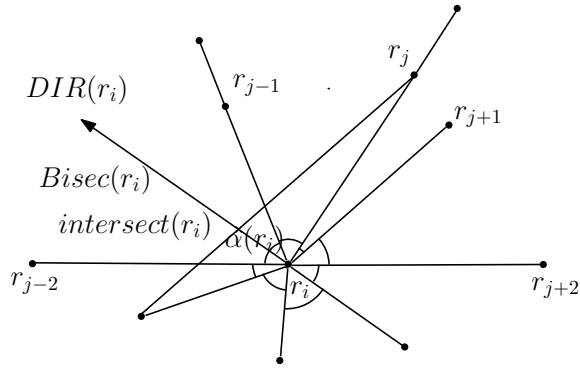
Figure 4: Examples of $\Gamma(\mathbf{r_i})$, $\alpha(\mathbf{r_i})$, Bisec($\mathbf{r_i}$), intersect($\mathbf{r_i}$)

- **$\Gamma(\mathbf{r_i}$:)** Set of angles $\angle r_j r_i r_k$ where r_k and r_j are two consecutive vertices of $STR(r_i)$ (Figure 4) .

- **$\alpha(\mathbf{r_i})$:** Maximum of $\Gamma(r_i)$ if maximum value of $\Gamma(r_i)$ is less than π otherwise the 2nd maximum of $\Gamma(r_i)$. The tie, if any, is broken arbitrarily (Figure 4).

- **Bisec($\mathbf{r_i}$)** : Bisector of $\alpha(r_i)$. Note that $Bisec(r_i)$ is a ray from r_i towards the angle of consideration (Figure 4).

- **DIR($\mathbf{r_i}$)** : The direction of $Bisec(r_i)$. We say that $DIR(r_i)$ lies on that side of any straight line where infinite end of $DIR(r_i)$ lies (Figure 4).

- **intersect($\mathbf{r_i}$)** : We look at the intersection points of $Bisec(r_i)$ and $\mathcal{L}_{jk}$, $\forall r_j, r_k \in \mathcal{V}(r_i)$. The intersection point closest to r_i is denoted by $intersect(r_i)$ (Figure 4).

- **$\Gamma'(\mathbf{r_i})$:** Set of angles $\angle r_{i-1} r_j r_i$ and $\angle r_i r_j r_{i+1}$, $\forall r_j \in \mathcal{V}(r_i)$, where r_{i-1} and r_{i+1} are the two neighbors of r_i on $STR(\mathcal{V}(r_j))$ (Figure 5).
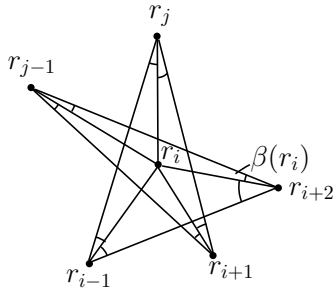

Figure 5: Examples of $\Gamma'(\mathbf{r_i})$, $\beta(\mathbf{r_i}) = \angle \mathbf{r_{j-1} r_{i+2} r_i}$

- **$\beta(r_i)$:** Minimum of $\Gamma(r_i) \cup \Gamma'(r_i)$ (Figure 5).

- **$\theta(r_i)$:** $\frac{\beta(r_i)}{n^2}$.

- **$d(r_i)$:** Distance between r_i and $intersect(r_i)$.

- **D($\mathbf{r_i}$)** : Distance between r_i and the robot nearest to it.

- **$\Delta(r_i)$:** $min\{\frac{d(r_i)}{n^2}, D(r_i)Sin(\theta(r_i))\}$.

- **$\hat{r}_i$:** The point on $Bisec(r_i)$, $\Delta(r_i)$ distance apart from r_i (Figure 6).

- **$C(r_i)$:** The circle of radius $\Delta(r_i)$ centered at r_i. Note that $\hat{r}_i$ always lies on $C(r_i)$ (Figure 6).

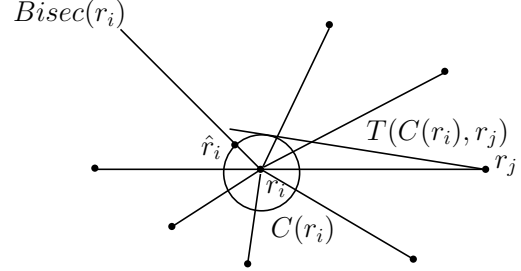- **$T(C(r_i), r_j)$:** Any one of the tangential points of the tangents drawn to $C(r_i)$ from r_j (Figure 6).

Figure 6: Examples of $\mathbf{C(r_i)}$, $\hat{r}_i$, $\mathbf{T(C(r_i), r_j)}$

We classify the robots in $\mathcal{R}$ depending upon their positions with respect to $\mathcal{CH}(\mathcal{R})$ (the convex hull of $\mathcal{R}$), as below:

- **External vertex robots (R_{EV}):** A set of robots lying on the vertices of $\mathcal{CH}(\mathcal{R})$. These robots do not obstruct the visibility of any robot in $\mathcal{R}$ and hence they do not move during whole execution of the algorithm. Note that, if r_i lies outside of $STR(r_i)$, then r_i is an external vertex robot.

- **External edge robots (R_{EE}):** A set of robots lying on the edges of $\mathcal{CH}(\mathcal{R})$. These robots either block the visibility of external vertex robots or other robot edge robots. Note that, if r_i lies on an edge of $STR(r_i)$, then r_i is an external edge robot.

- **Internal robots (R_I):** A set of robots lying inside the $\mathcal{CH}(\mathcal{R})$. Note that, if r_i lies within $STR(r_i)$, r_i is an internal robot.

3. ALGORITHM FOR MAKING OF GENERAL POSITION

Consider initially robots in $\mathcal{R}$ are not in general position. Our objective is to move the robots in $\mathcal{R}$ in such a way that after a finite number of movements of the robots in $\mathcal{R}$, it will be in general position. In order to do so, our approach is to move the robots which create visual obstructions to the other robots. If a robot r_i lies between two other robots, say r_p and r_q such that r_i, r_p and r_q are in straight line, then r_i is selected for movement. The destination of r_i, say $T(r_i)$, is computed in such a way that, there always exists a $r_j \in \mathcal{R}$ (where r_j does not have full visibility), such that when r_i moves, the cardinality of the set of visible robots of r_j increases. Since, the number of robots are finite, the number of robots having partial visibility, is also finite. Our algorithm assures that at each round at-least one robot with partial visibility will have full visibility. This implies that in finite number of rounds all robots will achieve full visibility, hence, the robots will be in general position in finite time.

3.1 Computing the destinations of the robots

A collinear middle robot is selected to move from its position. A robot finds its destination for movement using algorithm $ComputeDestination(r_i)$. A robot r_i, selected for moving, moves along the bisector of the minimum angle created at r_i by the robots in $\mathcal{V}(r_i)$. The destination is chosen in such a way that r_i will not block the vision of any $r_j \in \mathcal{V}(r_i)$, where the vision of r_j was not initially blocked by r_i, throughout the paths towards its destination. Each movement of r_i breaks at least one initial collinearity.

Algorithm 1: ComputeDestination()

Input: $r_i \in R$ with $COL(r_i) \neq \phi$.

Output: a point on $Bisec(r_i)$.

1. Compute $\alpha(r_i)$, $Bisec(r_i)$, $\beta(r_i)$, $\theta(r_i)$, $D(r_i)$,

2. **Case 1:** $\beta(r_i) \neq 0$,
   $\Delta(r_i) \leftarrow min\{\frac{d(r_i)}{n^2}, D(r_i)Sin(\theta(r_i))\}$

3. **Case 2:** $\beta(r_i) = 0$,
   $\Delta(r_i) \leftarrow D(r_i)$

4. Compute the point $\hat{r}_i$ on $Bisec(r_i)$, $\Delta(r_i)$ distance apart from r_i;

5. return $\hat{r}_i$;

Proof of Correctness of algorithm ComputeDestination().

Correctness of the algorithm is established by following observations, lemmas.
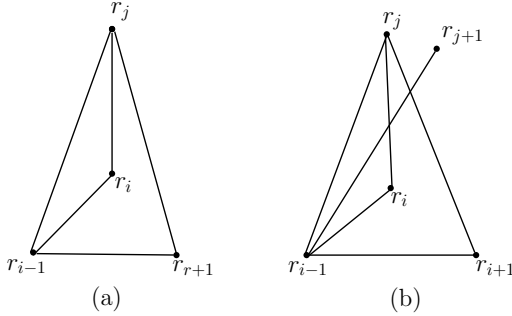

Figure 7: An example for lemma 1

LEMMA 1. $\beta(r_i) \leq \frac{\pi}{3}$.

PROOF. If all the robots lie on a straight line, then $\beta(r_i) = 0$. Suppose there are at least three non-collinear robots. For three robots forming a triangle, $\beta(r_i)$ is maximum when the triangle is equilateral. For all other cases, consider the triangle formed by r_i, r_j and r_{i-1} where r_j is any robot in $\mathcal{V}(r_i)$ and r_{i-1} is a neighbor of r_i on $STR(\mathcal{V}(r_j))$. If r_j is also a neighbor of r_i on $\mathcal{V}(r_{i-1})$ (Figure 7(a)), then $\angle r_i r_j r_{i-1}$ and $\angle r_i r_{i-1} r_j$ are in $\Gamma'(r_i)$ and either $\angle r_j r_i r_{i-1}$ or an angle less than it is in $\Gamma(r_i)$. On the other hand, if r_j is not a neighbor of r_i on $\mathcal{V}(r_{i-1})$ (Figure 7(b)), then instead of $\angle r_i r_{i-1} r_j$, an angle less than it, is in $\Gamma'(r_i)$. In all cases, $\beta(r_i)$ is less than

the minimum of the angles of the triangle formed by r_i, r_j and r_{i-1}. Hence, $\beta(r_i) \leq \frac{\pi}{3}$.

$\square$

Observation 1. *Maximum value of $DISP(r_i r_j)$, denoted by $Max(DISP(r_i r_j)$, is attained when $\hat{r}_i$ coincides with one of the tangential points $T(C(r_i), r_j)$.*

LEMMA 2. *For any r_i, $DISP(r_i r_j) \leq \theta(r_i)\ \forall\ r_j$.*

PROOF. Let r_j be a robot in $\mathcal{V}(r_i)$ and r_k a robot closest to r_i. By observation 1, maximum values of $DISP(r_i r_j)$ and $DISP(r_i r_k)$ are attained at tangential points $T(C(r_i), r_j)$ and $T(C(r_i), r_k)$ respectively. Hence, $DISP(r_i r_j)$ is less than $\frac{\pi}{2}$ for all j. By definition,

$$\frac{\Delta(r_i)}{|\overline{r_i r_k}|} = sin(max(DISP(r_i r_k)))$$
$$\leq sin(\theta(r_i)) \qquad (1)$$

Again,

$$\frac{\Delta(r_i)}{|\overline{r_i r_j}|} = sin(max(DISP(r_i r_j))) \qquad (2)$$

Since $|\overline{r_i r_k}| < |\overline{r_i r_j}|$, from (1) and (2) we have,

$$sin(max(DISP(r_i r_j))) \leq sin(\theta(r_i)). \qquad (3)$$

$DISP(r_i r_j)$ and $\theta(r_i)$ are in $[0, \frac{\pi}{2})$ (by lemma 1) and *sine* is an increasing function in $[0, \frac{\pi}{2}]$. From (3) we conclude,

$$DISP(r_i r_j) \leq \theta(r_i)$$

$\square$

Suppose a robot $r_i \in \mathcal{R}$ moves according to our algorithm. We claim that it will never become collinear with any two robots r_j and r_k in $\mathcal{R}$ where r_i, r_j and r_k are not collinear initially. Now we state arguments to prove our claim.

Observation 2. *Let ABC be a right-angled triangle with $\angle ABC = \frac{\pi}{2}$. Let D be a point on the side AC such that $|DC| \leq \frac{1}{2}|AC|$. Then,*

$$\angle BDA \leq 2\angle ACB.$$

LEMMA 3. *Suppose r_i and r_j move to new positions $\hat{r}_i$ and $\hat{r}_j$ in at most one computation cycle. Let ϕ be the angle between $\mathcal{L}_{r_i r_j}$ and $\mathcal{L}_{\hat{r}_i \hat{r}_j}$ i.e., $\phi = \angle r_i c \hat{r}_i$ where c is the intersection point between $\mathcal{L}_{r_i r_j}$ and $\mathcal{L}_{\hat{r}_i \hat{r}_j}$. Then,*

$$\phi < 2\ Max\ \{\theta(r_i), \theta(r_j)\}$$

PROOF. If any one r_i and r_j moves, then lemma is trivially true. Suppose both of them move once.

Case 1:
Suppose r_i and r_j move synchronously. Without loss of generality, let $\Delta(r_i) \geq \Delta(r_j)$.

- **Case 1.1:**
  Suppose $DIR(r_i)$ and $DIR(r_j)$ lie in the opposite sides of $\mathcal{L}_{r_i r_j}$ (Figure 8). In view of observation 1, Max$\{\phi\}$, the maximum value of ϕ, is attained when $\mathcal{L}_{\hat{r}_i \hat{r}_j}$ is a common tangent to $C(r_i)$ and $C(r_j)$. Let M be the middle point of $\overline{r_i r_j}$. If
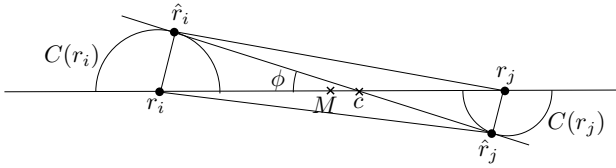
Figure 8: An example of case 1.1 for lemma 3

$C(r_i)$ is strictly larger than $C(r_j)$, c is closer to r_j than r_i. If they are equal, c coincides with M. Consider the right-angled triangle $\triangle r_i \hat{r}_i r_j$. By observation 2,

$$
\begin{aligned}
\phi &\le Max\{\phi\} \\
&\le 2DISP(r_i r_j) \\
&< 2Max\{DISP(r_i r_j)\} \\
&\le 2\theta(r_i)
\end{aligned}
$$

    – **Case 1.2:**
If $DIR(r_i)$ and $DIR(r_j)$ lie in the same side of $\mathcal{L}_{r_i r_j}$ (Figure 9), $Max\{\phi\}$ is attained when $\mathcal{L}_{\hat{r}_i \hat{r}_j}$ is a tangent to $C(r_i)$ from the point c and c coincides with the closest point of $C(r_j)$ from r_i. Then following same argument as in case-1, we have the proof.
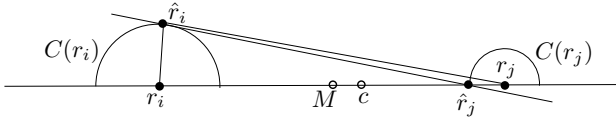

Figure 9: An example of case 1.2 for lemma 3

• **Case 2:**
Suppose r_i and r_j move asynchronously. Suppose r_i is moving and is at r_i' when r_j takes the snapshot of its surroundings to compute the value of $\Delta(r_j)$. Since r_i has already computed the value of $\Delta(r_i)$ and computation of Δ values of r_i and r_j are independent, the proof follows from the same arguments as in case 1. In this case the value of $\Delta(r_j)$ may be different from the value in case 1.

$\square$

LEMMA 4. *Suppose two robots r_i and r_j move to $\hat{r}_i$ and $\hat{r}_j$ respectively in at most one movement. Then*

$$Max\{DISP(r_i \hat{r}_j), DISP(r_j \hat{r}_i)\} < 2Max\{\theta(r_i), \theta(r_j)\}.$$

PROOF. Follows from observation 2 and lemma 3 (Figure 10). $\square$

LEMMA 5. *If r_i, r_j and r_k are not collinear and mutually visible to each other, then during the whole execution of the above algorithm, they never become collinear.*
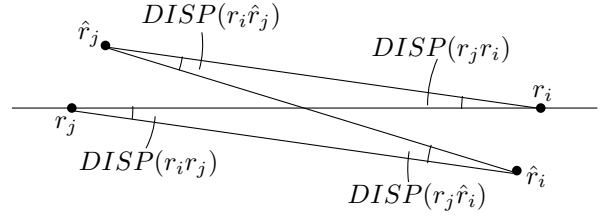
PROOF. We have the following cases,

Figure 10: An example for lemma 4

• **Case 1 (Only one robot moves):**
Without loss of generality, suppose r_j, r_k stand still and r_i moves. If $DIR(r_i)$ does not intersect $\mathcal{L}_{r_j r_k}$ (Figure 11(a)), then the claim is trivially true.

Suppose $DIR(r_i)$ intersects $\mathcal{L}_{r_j r_k}$ (Figure 11(b)). Since distance traversed by r_i is bounded above by $\frac{d(r_i)}{n^2}$, r_i can not reach $\mathcal{L}_{r_j r_k}$ and r_i, r_j and r_k will not become collinear.
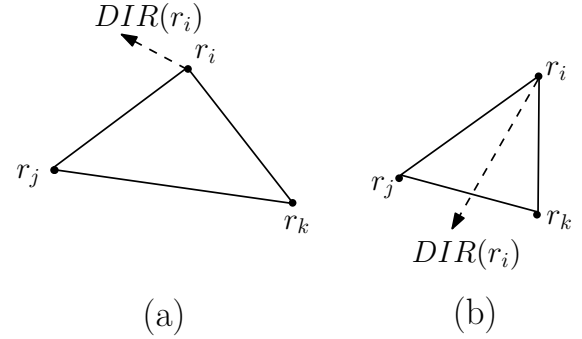

Figure 11: An example of case 1 for lemma 5

• **Case 2 (Two of the robots move):**
Without loss of generality, suppose r_i and r_j move while r_k remains stationary. This case would be feasible only if $n \ge 4$.

    – **Case 2.1:**
Suppose r_i and r_j move synchronously. Then by lemma 2,

$$DISP(r_i r_k) \le \frac{\angle r_i r_k r_j}{n^2} \qquad (4)$$

And

$$DISP(r_j r_k) \le \frac{\angle r_i r_k r_j}{n^2} \qquad (5)$$

From equation 4 and 5

$$DISP(r_i r_k) + DISP(r_j r_k) < \angle r_i r_k r_j \qquad (6)$$

The minimum value of $DISP(r_i r_k) + DISP(r_j r_k)$ for which r_i, r_j and r_k could become collinear is $\angle r_i r_k r_j$. In view of equation (6), we conclude that r_i, r_j and r_k would never become collinear.

    – **Case 2.2:**
Suppose that r_i is in motion and is at $\hat{r}_i'$ when r_j computes the value of $\Delta(r_j)$. If $\hat{r}_i'$ and r_j lie in opposite sides of $\mathcal{L}_{r_i r_k}$ (Figure 12(a)), then
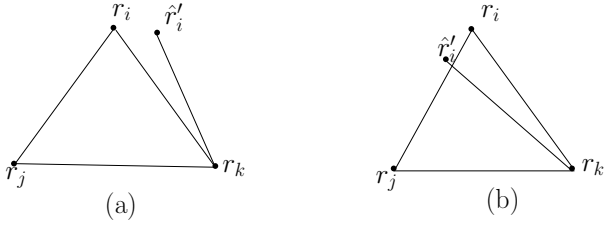
Figure 12: An example of case 2.2 for lemma 5

$$\Delta(r_j) \leq \frac{1}{n^2} dist(r_j, \mathcal{L}_{r_k \hat{r}_i{}'})$$

which implies that r_j can not reach $\mathcal{L}_{r_i r_k}$ when r_i reaches its destination and hence the lemma. Suppose $\hat{r}_i{}'$ and r_j lie in same side of $\mathcal{L}_{r_i r_k}$ (Figure 12(b)). Then we have,

$$DISP(r_i r_k) \leq \frac{\angle \hat{r}_i{}' r_k r_j}{n^2}$$
$$< \frac{\angle \hat{r}_i r_k r_j}{n^2}$$

Lemma follows from the same arguments as used in Case 2.1.

Consider the case: suppose r_j takes the snapshot at time t and moves to its destination at time t'. In between times t and t', suppose r_i has made at most $\frac{n-1}{2}$ moves (we shall prove in case 3.2 that number of movements of any robot is bounded above by $\frac{n-1}{2}$). If r_i moves towards r_j, after $\frac{n-1}{2}$ moves, we would have

$$DISP(r_i r_j) < (1 - \frac{1}{n^2})^{\frac{n-1}{2}} \angle r_i r_k r_j$$

which is less than $(1 - \frac{1}{n^2})\angle r_i r_k r_j$. Hence equation (6) is satisfied in this case and we have the proof of the lemma. If r_i moves away from r_j, then there is nothing to prove.

- **Case 3 (All three robots move):**

  - **Case 3.1:**
    Suppose r_i, r_j and r_k move synchronously.

  - **Case 3.1.1:**
    Suppose $\mathcal{L}_{\hat{r}_i \hat{r}_j}$ intersects $\mathcal{L}_{r_i r_j}$ at an angle $\phi > 0$ (Figure 13).
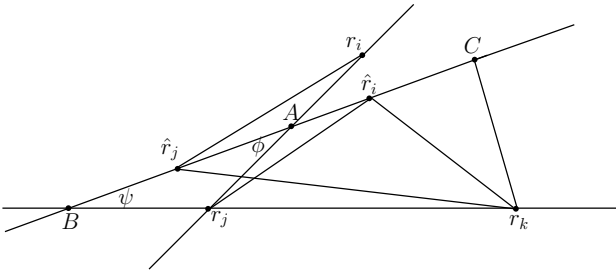

Figure 13: An example of case 3.1.1 for lemma 5

By lemma 3,

$$\phi < 2Max\{\theta(r_i), \theta(r_j)\}$$
$$\leq \frac{2}{n^2} \angle r_i r_j r_k \qquad (7)$$

In $\triangle ABr_j$,

$$\psi = \angle r_i r_j r_k - \phi$$
$$> \angle r_i r_j r_k - \frac{2}{n^2} \angle r_i r_j r_k$$
$$= \frac{n-2}{n^2} \angle r_i r_j r_k$$
$$\geq \frac{3}{5^2} \angle r_i r_j r_k \qquad (8)$$

Now r_i, r_j and r_k would be collinear only if

$$DISP(r_k B) = \psi \qquad (9)$$

From lemma 4,

$$DISP(r_k B) < DISP(r_k \hat{r}_j)$$
$$< 2Max\{\theta(r_i), \theta(r_j)\}$$
$$\leq \frac{2}{5^2} \angle r_i r_j r_k \qquad (10)$$

Equations 8, 9 and 10 imply that r_i, r_j and r_k do not become collinear.

  - **Case 3.1.2:**
    Suppose $\mathcal{L}_{\hat{r}_i \hat{r}_j}$ and $\mathcal{L}_{r_i r_j}$ are parallel i.e., $\phi = 0$ which implies that $\psi = \angle r_i r_j r_k$ (Figure 14). Let $Bisec(r_k)$ intersect $\mathcal{L}_{r_i r_j}$ at P and $|\overline{r_k P}| = l$. Since $\Delta(r_k) \leq |\overline{r_i r_j}|sin(\frac{\angle r_i r_j r_k}{n^2})$ and $n \geq 5$,
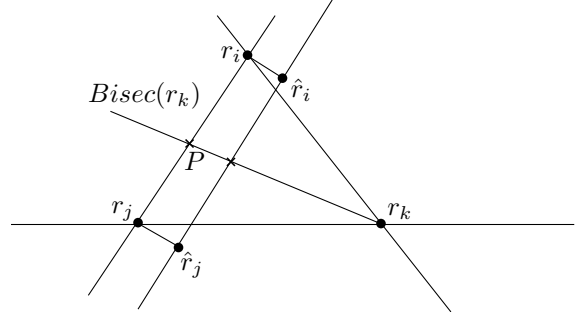

Figure 14: An example of case 3.1.2 for lemma 5

$$l - \Delta(r_k) \geq |\overline{r_i r_j}|sin(\angle r_i r_j r_k) - \Delta(r_k)$$
$$\geq |\overline{r_i r_j}|sin(\angle r_i r_j r_k) - |\overline{r_i r_j}|sin(\frac{\angle r_i r_j r_k}{n^2})$$
$$\geq |\overline{r_i r_j}|(sin(\angle r_i r_j r_k) - sin(\frac{\angle r_i r_j r_k}{5^2}))$$
$$> |\overline{r_i r_j}|sin(\frac{\angle r_i r_j r_k}{5^2}) \qquad (11)$$

$\Delta(r_i)$ and $\Delta(r_j)$ are bounded above by $|\overline{r_i r_j}|sin (\frac{\angle r_i r_j r_k}{5^2})$. Hence by equation (11), r_i and r_j and r_k do not become collinear.

- **Case 3.2:**
  Suppose r_i, r_j and r_k move asynchronously. The main problem in this case is the following scenario: suppose

r_j or r_k takes the snapshot at time t_j or t_k respectively and starts moving to its computed destination at time t'_j or t'_k respectively. Suppose the configuration has been changed in between the times due to the movements of the other robots. Then the corresponding Δ value of r_j or r_k is not consistent w.r.t. the current configuration. We have to show that this would not create any problem for our algorithm. The main idea of proof in this case is that we have to estimate the maximum amount of inclination of $\mathcal{L}_{r_i r_j}$ towards r_k between the times r_j or r_k takes the snapshot of surroundings and it reaches the destination. So, in the following proofs we only consider the scenarios (as in the case 3.1.1. and case 3.1.2) in which there are possibilities of maximum reduction in the $\angle r_i r_j r_k$, which depicts the inclination of $\mathcal{L}_{r_i r_j}$ towards r_k. Note that the inclination of $\mathcal{L}_{r_i r_j}$ towards r_k is maximum when both r_i and r_j move synchronously. So, we only prove the case when r_k holds the old value of Δ.

- **Case 3.2.1**
  Suppose r_k holds the old value of Δ w.r.t. to the current configuration. Suppose r_i and r_j are at r_i^0 and r_j^0 respectively when r_k takes the snapshot at time t_k. Suppose till t'_k, r_i and r_j move x and x' times respectively. Note that initially r_i and r_j can be collinear with $n-1$ robots and to remove these collinearity they have to move at most $\frac{n-1}{2}$ times if they do not create any new collinearity (this bound is obtained by considering the degenerate case i.e., when all the robots are collinear initially).
  First we prove that x and x' are bounded above by $\frac{n-1}{2}$. To prove this we show that r_i and r_j do not create any new collinearity while moving. We prove this for arbitrary robots. Suppose some robot r_s, while moving, creates a new collinearity with r_l and r_m for the first time during the execution of our algorithm (Figure 15). Then either one of r_l and r_m or both
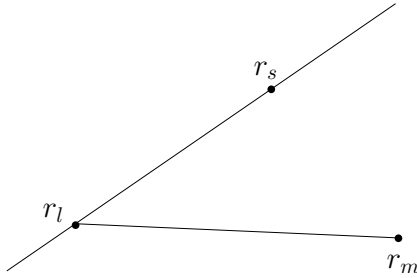

Figure 15: An example of case 3.2.1 for lemma 5

have Δ values w.r.t. old configurations. As stated earlier we only prove the case in which only one robot, say r_m, has old Δ value. r_m computes $\Delta(r_m)$ at the time t_m i.e.,

$$\Delta(r_m) \leq \frac{1}{n^2} \angle r_s r_l r_m.$$

Suppose r_m does not move till time t'_m. The number of times r_s and r_l move to break the initial collinearities before time t'_m is upper bounded by $\frac{n-1}{2}$. r_m would become collinear with r_s and r_l when $\mathcal{L}_{r_s r_l}$ would be inclined enough towards r_m so that by moving a $\Delta(r_m)$

amount it would reach this straight line. We try to estimate the inclination of $\mathcal{L}_{r_s r_l}$ towards r_m (which is depicted by the angle ψ as in the case 3.1.1. and by the displacement of $\mathcal{L}_{r_s r_l}$ towards r_m as in the case 3.1.2.) after $\frac{n-1}{2}$ number of movements of r_s and r_l (note that we have consider the over estimated value of the number of movements of r_s and r_l). As computed in the case 3.1.1, after first movement,

$$\psi > (1 - \frac{1}{n^2}) \angle r_s r_l r_m$$

and $\angle r_s r_l r_m$ will become at most $(1 + \frac{1}{n^2}) \angle r_s r_l r_m$. By the same repeated arguments, we can say that after d movements

$$\psi > (1 - \frac{1}{n^2})^d \angle r_s r_l r_m$$

which is strictly greater than $\frac{1}{n^2} \angle r_s r_l r_m$ for $d \leq \frac{n-1}{2}$. This contradicts the fact that r_s creates collinearity with r_l and r_m. For the scenario same as the case 3.1.2., we have,

$$|\overline{r_l r_m}| sin(\angle r_s r_l r_m) - \frac{n-1}{2} |\overline{r_l r_m}| sin(\frac{\angle r_s r_l r_m}{n^2}) > \\ |\overline{r_l r_m}| sin(\frac{\angle r_s r_l r_m}{n^2}) \quad (12)$$

This also contradicts the fact that r_s creates collinearity with r_l and r_m. Hence, we conclude that r_s would not become collinear with r_l and r_m.
In the above proof, we replace r_s, r_l and r_m by r_i, r_j and r_k respectively to conclude that r_i would not become collinear with r_j and r_k during the whole execution of our algorithm.

$\square$

LEMMA 6. *Consider any two robots r_i and r_j. r_i does not cross $Bisec(r_j)$.*
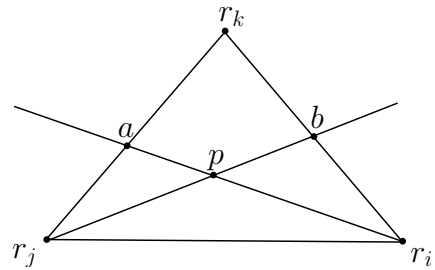

Figure 16: An example for lemma 6

PROOF. If $Bisec(r_i)$ and $Bisec(r_j)$ do not intersect, then there is nothing to prove. Suppose $Bisec(r_i)$ and $Bisec(r_j)$ intersect at a point p (Figure 16). If at least one of $intersect(r_i)$ and $intersect(r_j)$ is closer to r_i and r_j respectively than p, then we are done. Else $\alpha(r_i)$ and $\alpha(r_j)$ are angle of same triangle $\triangle r_i r_j r_k$ for some $r_k \in \mathcal{R}$ i.e, $\alpha(r_i) = \angle r_k r_i r_j$ and $\alpha(r_i) = \angle r_k r_j r_i$. In $\triangle r_i r_j r_k$, let $Bisec(r_i)$ and $Bisec(r_j)$ intersect $\overline{r_j r_k}$ and $\overline{r_i r_k}$ at a and b respectively. Here $n > 5$.

In $\triangle ar_jp$,

$$|\overline{ap}| = sin(\frac{\angle r_k r_j r_i}{2})\frac{|\overline{r_j a}|}{sin(\angle apr_j)} \qquad (13)$$

In $\triangle pr_i r_j$,

$$|\overline{pr_i}| = sin(\frac{\angle r_k r_j r_i}{2})\frac{|\overline{r_i r_j}|}{sin(\angle \pi - apr_j)}$$
$$= sin(\frac{\angle r_k r_j r_i}{2})\frac{|\overline{r_i r_j}|}{sin(\angle apr_j)} \qquad (14)$$

From equation 13 and 14,

$$\frac{|\overline{ap}|}{|\overline{pr_i}|} = \frac{|\overline{r_j a}|}{|\overline{r_i r_j}|} \qquad (15)$$

Since $|\overline{r_j a}| < |\overline{r_i r_j}|$, $|\overline{ap}| < |\overline{pr_i}|$ which implies,

$$\Delta(r_i) < \frac{|\overline{r_i a}|}{5^2}$$
$$< |\overline{pr_i}|.$$

Hence r_i can not cross $Bisec(r_j)$. Similarly, r_j can not cross $Bisec(r_i)$. $\square$

LEMMA 7. *Suppose, for any robot $r_i \in \mathcal{R}$, $r_k \notin \mathcal{V}(r_i)$. Then during the whole execution of the algorithm r_i will not block the vision between r_j and r_k where $r_j \in \mathcal{V}(r_k)$.*
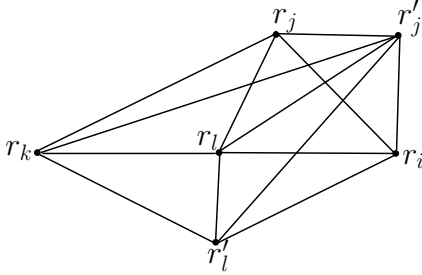

Figure 17: An example for lemma 7

PROOF. Let $r_j \in \mathcal{V}(r_i) \cap \mathcal{V}(r_k)$. Suppose r_l be the nearest robot of r_i such that r_k lie on $\mathcal{L}_{r_i r_l}$ (Figure 17). If $Bisec(r_i)$ does not intersect $\overline{r_j r_l}$, there is no possibility that r_i will block the vision between r_j and r_k. Let $Bisec(r_i)$ intersect $\overline{r_j r_l}$. Then r_j is one of the immediate neighbor of r_l on $STR(\mathcal{V}(r_i))$. Let r'_j and r'_l be the other immediate neighbors of r_j and r_l respectively on $STR(\mathcal{V}(r_i))$. First we prove that r_i will always lie on the same side of $\mathcal{L}_{r_j r_l}$ as it is initially even if r_i, r_j, r_k and r_j move. By lemma 6 and the observation that the movements of r_i, r_j, r_l are bounded by the edges and chords of the polygon formed by $\{r_j, r_l, r'_l, r_i, r'_j\}$, we conclude r_i never crosses the line $\mathcal{L}_{r_j r_l}$. To block the vision between r_k and r_j, r_i has to move on the line segment $\overline{r_k r_j}$. Since r_i and line segment $\overline{r_j r_k}$ lies on different sides of $\mathcal{L}_{r_j r_l}$, r_i will never block the vision between r_k and r_j. Let $r_j \notin \mathcal{V}(r_i)$. Then there is a robot r_m which creates visual obstruction between r_i and r_j. Now the movement of r_i is bounded by the line $\mathcal{L}_{r_l r_m}$ and hence the lemma. $\square$

LEMMA 8. *If at any time t, $r_j \in \mathcal{V}(r_i)$, then at $t'(> t)$, $r_j \in \mathcal{V}(r_i)$ even if r_i changes its position.*

PROOF. The proof is immediate from 5 and 7. $\square$

LEMMA 9. *Cardinality of $\mathcal{V}(r_i)$ is strictly increasing.*

PROOF. Lemma 5, 7 and 8 imply the proof. $\square$

LEMMA 10. *There exist at least two robots $r_j, r_k \in \mathcal{R}$ for which $\mathcal{V}(r_j)$ and $\mathcal{V}(r_j)$ increase whenever r_i changes its position.*

PROOF. r_i moves whenever r_i is collinear with at least one pair of robots, (r_j, r_k), and r_i lies in between those robots. If r_j and r_k do not move then $\mathcal{V}(r_j)$ and $\mathcal{V}(r_k)$ increase whenever r_i moves because no robot can reach $\overline{r_j r_k}$ due to the facts stated in lemma 5 and 7. When either r_j or r_k or both r_i and r_k moves, one member of $COL(r_j)$ and one member of $COL(r_k)$ can see each other. Hence the lemma. $\square$

3.2 Moving the robots to obtain general position

Next we will discuss the algorithm $MakeGenaralPosition()$, by which the robots in $\mathcal{R}$ move to obtain full visibility. The robots in R_I which create obstacle to other robots and the robots in R_{EE} are eligible for movement by this algorithm. The robots compute destinations using $ComputeDestination()$ and move towards it. The robots keep on executing the algorithm till there exist no three collinear robots in $\mathcal{R}$.

Algorithm 2: MakeGenaralPosition()

Input: $\mathcal{R}$, a set of robots with their positions.

Output: $\hat{\mathcal{R}}$, which is in general position.

while $r_i \in R_{EE} \lor (r_i \in R_I \land COL(r_i) \neq \phi)$ **do**

    1. $T(r_i) \leftarrow ComputeDestination(r_i)$;

    2. Move to $T(r_i)$;

    3. Compute $COL(r_i)$;

Proof of Correctness of algorithm MakeGenaralPosition().

The algorithm assures that the robot will form general position in finite number of movements. The termination of the algorithm is established by following observation and lemmas.

Observation 3. *ComputeDestination is not executed by a robot $r_l \in R$ if $r_l \in R_{EV} \lor (R_I \land COL(r_l) = \phi)$.*

LEMMA 11. *$COL(r_i)$ will be ϕ in finite time.*

PROOF. In the initial configuration the number of robots in $COL(r_i)$ is upper bounded by $n - 1$. During the whole execution of our algorithm no new collinearity is created and for each iteration cardinality of $COL(r_i)$ is reduced by at least two. Hence after at most $\frac{n-1}{2}$ number of iterations of the while loop in the above algorithm, $COL(r_i)$ will become null. $\square$

LEMMA 12. *$\forall r_i, \mathcal{V}(r_i)$ will be $(n-1)$ in finite number of execution of the cycle.*

PROOF. Let $\eta = |\bigcup_{i=1}^{n} \mathcal{V}(r_i)|$. The algorithm for a robot r_i terminates whenever $|\mathcal{V}(r_i)|$ reaches the value $n-1$. Hence the algorithm for all robots terminates when $\eta = \frac{n(n-1)}{2}$ which is a finite integer. By lemma 9 and 10 the value of η increases whenever any robot moves. Hence after finite number of execution cycles η reaches its maximum value $\frac{n(n-1)}{2}$. $\square$

From the above results, we can conclude the following theorem:

Theorem 1. *A set of asynchronous, oblivious robots (initially not in general position) without agreement in common chirality, can form general position in finite time.*

4. CONCLUSION

In this paper we have presented an algorithm for obtaining general position by a set of autonomous, homogeneous, oblivious, asynchronous robots having no common chirality. The algorithm assures the robots to have collision free movements. Another important feature of our algorithm is that the convex hull made by the robots in initial position, remains intact both in location and size. In other words, the robots do no go out side the convex hull formed by them. This feature can help in many subsequent pattern formations which require to maintain the location and size and of the pattern.

Once the robots obtain general position, the next job could be to form any pattern maintaining the general position. Most of the existing pattern formation algorithms have assumed that the robots are see through. Thus, designing algorithms for forming patterns by maintaining general position of the robots, may be a direct extension of this work.

5. REFERENCES

[1] C. Agathangelou, C. Georgiou, and M. Mavronicolas. A distributed algorithm for gathering many fat mobile robots in the plane. In *Proceedings of the 32nd ACM Symposium on Principles of Distributed Computing (PODC)*, 250–259, 2013.

[2] G. Antonio Di Luna, P. Flocchini, S. Gan Chaudhuri, N. Santoro, and G. Viglietta. Robots with Lights: Overcoming Obstructed Visibility Without Colliding In *Proc. 16th International Symposium on Stabilization, Safety, and Security of Distributed Systems (SSS'14)*, to appear.

[3] G. Antonio Di Luna, P. Flocchini, F. Poloni,, N. Santoro, and G. Viglietta. The Mutual Visibility Problem for Oblivious Robots. In *Proc. 26th Canadian Conference on Computational Geometry (CCCG'14)*, to appear.

[4] K. Bolla, T. Kovacs, and G.Fazekas. Gathering of fat robots with limited visibility and without global navigation. In *Int. Symp. on Swarm and Evolutionary Comp.*, 30–38, 2012.

[5] R.Cohen and D.Peleg. Local spreading algorithms for autonomous robot systems. *Theoretical Computer Science*, 399:71–82, 2008.

[6] J. Czyzowicz, L. Gasieniec, and A. Pelc. Gathering few fat mobile robots in the plane. *Theoretical Computer Science*, 410(6â7):481 – 499, 2009.

[7] S. Das, P. Flocchini, G. Prencipe, N. Santoro, and M. Yamashita. The power of lights: Synchronizing asynchronous robots using visible bits. In *Proceedings of the 32nd International Conference on Distributed Computing Systems (ICDCS)*, 506–515, 2012.

[8] S. Das, P. Flocchini, G. Prencipe, N. Santoro, and M. Yamashita. Synchronized dancing of oblivious chameleons. In *Proc. 7th Int. Conf. on FUN with Algorithms (FUN)*, 2014.

[9] A. Efrima and D. Peleg. Distributed models and algorithms for mobile robot systems. In *Proceedings of the 33rd International Conference on Current Trends in Theory and Practice of Computer Science (SOFSEM)*, 70–87, 2007.

[10] P. Flocchini, G. Prencipe, and N. Santoro. *Distributed Computing by Oblivious Mobile Robots*. Morgan & Claypool, 2012.

[11] P. Flocchini, N. Santoro, G. Viglietta, and M. Yamashita. Rendezvous of two robots with constant memory. In *Proceedings of the 20th International Colloquium on Structural Information and Communication Complexity (SIROCCO)*, 189–200, 2013.

[12] D. Peleg. Distributed coordination algorithms for mobile robot swarms: New directions and challenges. In *Proc. 7th Int. Workshop on Distr. Comp. (IWDC)*, 1–12, 2005.

[13] G. Viglietta. Rendezvous of two robots with visible bits. In *Proc. 9th Symp. on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics (ALGOSENSORS)*, 291–306, 2013.