Dynamic analysis of simultaneous adaptation of force, impedance and trajectory

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When carrying out tasks in contact with the environment, humans are found to concurrently adapt force, impedance and trajectory. Here we develop a robotic model of this mechanism in humans and analyse the underlying dynamics. We derive a general adaptive controller for the interaction of a robot with an environment solely characterised by its stiffness and damping, using Lyapunov theory.

I. System dynamics

The dynamics of a n-degree-of-freedom (n-DOF) robot in the operational space are given by

$$M(q)\,\ddot{x} + C(q,\dot{q})\,\dot{x} + G(q) = u + f \tag{1}$$

where x is the position of the robot and q the vector of joints angle. $M(q)$ denotes the inertia matrix, $C(q,\dot{q})\dot{x}$ the Coriolis and centrifugal forces, and $G(q)$ the gravitational force, which can be identified using e.g. nonlinear adaptive control [1]. u is the control input and f the interaction force.

In [2], we have described the control input u in two parts:

$$u = v + w\,, \tag{2}$$

with v to track the *reference trajectory* x_r by compensating for the robot's dynamics, i.e.

$$v = M(q)\,\ddot{x}_e + C(q,\dot{q})\,\dot{x}_e + G(q) - \Gamma\varepsilon \tag{3}$$

where

$$\dot{x}_e = \dot{x}_r - \alpha e\,, \quad e \equiv x - x_r\,, \quad \alpha > 0\,, \tag{4}$$

Γ a symmetric positive-definite matrix with minimal eigenvalue $\lambda_{\min}(\Gamma) \geqslant \lambda_\Gamma > 0$ and

$$\varepsilon \equiv \dot{e} + \alpha\, e \tag{5}$$

the *tracking error*. w is to adapt impedance and force in order to compensate for the unknown interaction dynamics.

II. Force and impedance adaptation

Suppose that the interaction force can be expanded as

$$f = F_0^* + K_S^*(x - x_0^*) + K_D^*\dot{x}\,, \tag{6}$$

where the force $F_0^*(t)$, stiffness $K_S^*(t)$ and damping $K_D^*(t)$ are feedforward components of the interaction force, $x_0^*(t)$ is the rest position of the environment visco-elasticity and all of these functions are unknown but periodic with T:

$$\begin{aligned} F_0^*(t+T) &\equiv F_0^*(t)\,, \quad K_S^*(t+T) \equiv K_S^*(t)\,, & (7)\\ K_D^*(t+T) &= K_D^*(t)\,, \quad x_0^*(t+T) = x_0^*(t)\,. & (8) \end{aligned}$$

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To simplify the analysis, we rewrite the interaction force as

$$f \equiv F^* + K_S^*\,x + K_D^*\,\dot{x} \tag{9}$$

where $F^* \equiv F_0^* - K_S^* x_0^*$ is also periodic with T. w in Eq.(2) is then defined as

$$w = -F - K_S x - K_D \dot{x} \tag{10}$$

where K_S and K_D are stiffness and damping matrices, respectively, and F is the feedforward force.

By substituting the control input u into Eq.(1), the closed-loop system dynamics are described by

$$M(q)\,\dot{\varepsilon} + C(q,\dot{q})\,\varepsilon + \Gamma\varepsilon = \widetilde{F} + \widetilde{K}_S\,x + \widetilde{K}_D\,\dot{x}\,, \tag{11}$$

$$\widetilde{F} \equiv F^* - F\,, \quad \widetilde{K}_S \equiv K_S^* - K_S\,, \quad \widetilde{K}_D \equiv K_D^* - K_D\,.$$

In this equation, we see that the feedforward force F, stiffness K_S and damping K_D ensure contact stability by compensating for the interaction dynamics. Therefore, the objective of force and impedance adaptation is to minimise these residual errors which can be carried out through minimising the cost function

$$\begin{aligned} J_c(t) \;\equiv\; & \frac{1}{2}\int_{t-T}^{t} \widetilde{F}^T Q_F^{-1}\widetilde{F} + \mathrm{vec}^T(\widetilde{K}_S)Q_S^{-1}\mathrm{vec}(\widetilde{K}_S) \\ & + \mathrm{vec}^T(\widetilde{K}_D)Q_D^{-1}\mathrm{vec}(\widetilde{K}_D)\,d\tau\,, \end{aligned} \tag{12}$$

where Q_F, Q_S and Q_D are symmetric positive-definite matrices, and $\mathrm{vec}(\cdot)$ stands for the column vectorization operation. This objective is achieved through the following update laws:

$$\begin{aligned} \delta F(t) &\equiv F(t) - F(t-T) \equiv Q_F[\varepsilon(t) - \beta(t)F(t)] \qquad (13)\\ \delta K_S(t) &\equiv K_S(t) - K_S(t-T) = Q_S[\varepsilon(t)x(t)^T - \beta(t)K_S(t)] \\ \delta K_D(t) &\equiv K_D(t) - K_D(t-T) = Q_D[\varepsilon\,\dot{x}(t)^T - \beta(t)K_D(t)] \end{aligned}$$

where F, K_S and K_D are initialised as zero matrices/vectors with proper dimensions for $t \in [0,T)$.

Now that we have dealt with the interaction dynamics, stable trajectory control can be obtained by minimising the cost function

$$J_e(t) \equiv \frac{1}{2}\varepsilon(t)^T M(q)\,\varepsilon(t)\,. \tag{14}$$

Consequently, we use a combined cost function $J_{ce} \equiv J_c + J_e$ that yields concurrent minimisation of tracking error and control effort.

III. TRAJECTORY ADAPTATION

In a typical interaction task, the contact between the robot and the environment is maintained through a desired interaction force F_d. Assuming that there exists a desired trajectory x_d yielding F_d, i.e. from Eq.(6)

$$\begin{aligned} F_d &= F_0^* + K_S^*(x_d - x_0^*) + K_D^*\,\dot{x}_d \\ &= F^* + K_S^*\,x_d + K_D^*\,\dot{x}_d\,, \quad F^* = F_0^* - K_S^*\,x_0^*\,, \end{aligned} \tag{15}$$

we propose to adapt the reference x_r in order to track x_d. However, x_d is unknown as the parameters F^*, K_S^* and K_D^* in the interaction force are unknown. Nevertheless, we know that x_d is periodic with T as F^*, K_S^* and K_D^* are periodic with T and we also set F_d to be periodic with T.

In the following, we develop an update law to learn the desired trajectory x_d. First, we define

$$\xi_d \equiv K_S^*\,x_d + K_D^*\,\dot{x}_d\,, \quad \xi_r \equiv K_S\,x_r + K_D\,\dot{x}_r\,. \tag{16}$$

Then, we develop the following update law

$$\delta\xi_r(t) \equiv \xi_r - \xi_r(t-T) \equiv L^{-T}Q_r(F_d(t) - F(t) - \xi_r(t)) \tag{17}$$

where Q_r and L are positive-definite constant gain matrices. This update law minimises the error between ξ_d and ξ_r, which is described by the following cost function

$$J_r \equiv \frac{1}{2}\int_{t-T}^{t}(\xi_r - \xi_d)^T Q_r^T(\xi_r - \xi_d)\,d\tau\,. \tag{18}$$

Because of the coupling of adaptation of force and impedance and trajectory adaptation, we modify the adaptation of feedforward force Eq.(13) to

$$\delta F(t) \equiv Q_F[\varepsilon(t) - \beta(t)F(t) + Q_r^T\delta\xi_r(t)]\,. \tag{19}$$

As a result, update laws Eqs.(17) and (19) minimise the overall cost $J = J_c + J_e + J_r$ as shown in Appendix A.

Then, we obtain the update law for trajectory adaptation

$$\delta x_r \equiv x_r(t) - x_r(t-T) \tag{20}$$

by solving

$$\delta\xi_r = K_S\,\delta x_r + K_D\,\delta\dot{x}_r = K_S\,\delta x_r + K_D\,\frac{d}{dt}(\delta x_r) \tag{21}$$

using $\delta\xi_r(t)$ from Eq.(17). According to the convergence of $\delta\xi_r$, K_S and K_D as shown in Appendix A, x_r will converge, as

$$\delta\xi_r - \xi_d = K_S\delta x_r + K_D\delta\dot{x}_r\,, \tag{22}$$

Upon convergence, the desired interaction force F_d is maintained between the robot and the environment according to Eq.(17). At the same time, the properties with adaptation of force and impedance are preserved which include trajectory tracking and control effort minimisation. However, from the analysis in Appendix A, we cannot draw the conclusion that F, K_S, K_D and x_r converge to F^*, K_S^*, K_D^* and x_d, respectively, which will require the condition of persistent excitation (PE), similar to classical adaptive control theory [3].

IV. DISCUSSION

A. No contact

In a special case when there is *no force applied by the environment* and F_d is also zero, the controller component w will converge to zero. According to the update law Eq.(17), the reference trajectory will not adapt, as expected.

B. No damping

If we *neglect the damping* component in the interaction force f of Eq.(9), the trajectory adaptation described by Eqs.(17) and (21) can be simplified to

$$\delta x_r = L^{-T}Q_r(F_d - F - K_S\,x_r) \tag{23}$$

Correspondingly, the update laws for force and impedance Eq.(13) needs to be modified as

$$\begin{aligned} \delta F &\equiv Q_F(\varepsilon - \beta F + Q_r^T\delta x_r)\,, \\ \delta K_S &\equiv Q_S(\varepsilon\,x^T - \beta K_S + x_r^T Q_r^T\delta x_r)\,. \end{aligned} \tag{24}$$

The stability analysis is similar to the case with damping and is briefly explained in Appendix B.

C. Force sensing

As in [2], force sensing is not required in the proposed framework, in contrast to traditional methods for surface following where the force feedback is used to regulate the interaction force [4].

In particular, in a first phase force and impedance adaptation is used to compensate for the interaction force from the environment. During this process, the unknown actual interaction force is estimated when the tracking error ε goes to zero as can be seen from Eq.(11): when $\varepsilon = 0$, we have

$$w = -f. \tag{25}$$

Using this estimated interaction force, then a desired force in Eq.(15) can be rendered by adaptation of the reference trajectory x_r.

In this sense, it is important to note that *trajectory adaptation should be conducted only when force and impedance adaptation takes effect, which guarantees compensation of the interaction force and tracking of the current reference trajectory*. Nevertheless, as shown in above stability analysis, adaptation of force, impedance and trajectory can be realised simultaneously.

This also suggests that a force sensor should be used if available, as force and impedance adaptation could then be replaced by force feedback. In this way, trajectory adaptation would not depend on the force estimation process and can in principle happen faster than force and impedance adaptation is needed. However, the potential advantages of a force sensor depends on the quality of the signal it could provide, its cost and the complexity of its installation and use.

V. Appendix

A. Proof for minimisation of overall cost J

Considering the definition of J_r in Eq. (18), we have

$$\begin{aligned}
&\delta J_r(t) \equiv J_r(t) - J_r(t-T) \\
= \;& \frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau) - \xi_d(\tau)]^T Q_r^T [\xi_r(\tau) - \xi_d(\tau)]\, d\tau \\
& -\frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau) - \xi_d(\tau)]^T Q_r^T [\xi_r(\tau-T) - \xi_d(\tau-T)]\, d\tau \\
& +\frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau) - \xi_d(\tau)]^T Q_r^T [\xi_r(\tau-T) - \xi_d(\tau-T)]\, d\tau \\
& -\frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau-T) - \xi_d(\tau-T)]^T Q_r^T \times \\
& \quad [\xi_r(\tau-T) - \xi_d(\tau-T)]\, d\tau \\
= \;& \frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau) - \xi_d(\tau)]^T Q_r^T \delta\xi_r(\tau)\, d\tau \\
& +\frac{1}{2}\int_{t-T}^{t} [\xi_r(\tau-T) - \xi_d(\tau-T)]^T Q_r^T \delta\xi_r(\tau)\, d\tau \\
= \;& \int_{t-T}^{t} [\xi_r - \xi_d - \frac{1}{2}\delta\xi_r]^T Q_r^T \delta\xi_r\, d\tau \quad (\text{as } \xi_d(t) = \xi_d(t-T)) \\
\leqslant \;& \int_{t-T}^{t} [Q_r(\xi_r(\tau) - \xi_d(\tau))]^T \delta\xi_r(\tau)\, d\tau\,. \qquad (26)
\end{aligned}$$

According to Eqs.(15) to (17), we rewrite this inequality as

$$\begin{aligned}
\delta J_r &\leqslant \int_{t-T}^{t} [Q_r(\xi_r - F_d + F + \widetilde{F})]^T \delta\xi_r\, d\tau \\
&= \int_{t-T}^{t} (-L^T\delta\xi_r + Q_r\widetilde{F})^T \delta\xi_r\, d\tau. \qquad (27)
\end{aligned}$$

Consider the difference between J_c of two consecutive periods

$$\begin{aligned}
&\delta J_c \equiv J_c - J_c(t-T) \qquad (28)\\
= \;& \frac{1}{2}\int_{t-T}^{t} [(\widetilde{F}^T Q_F^{-1}\widetilde{F} - \widetilde{F}^T(\tau-T) Q_F^{-1}\widetilde{F}(\tau-T)) \\
& +\text{tr}(\widetilde{K}_S^T Q_S^{-1}\widetilde{K}_S - \widetilde{K}_S^T(\tau-T) Q_S^{-1}\widetilde{K}_S(\tau-T) \\
& +(\widetilde{K}_D^T Q_D^{-1}\widetilde{K}_D - \widetilde{K}_D^T(\tau-T) Q_D^{-1}\widetilde{K}_D(\tau-T))]\, d\tau
\end{aligned}$$

where tr$(\cdot)$ stands for the trace of a matrix. We consider that

$$\begin{aligned}
& \widetilde{F}^T(\tau) Q_F^{-1}\widetilde{F}(\tau) - \widetilde{F}^T(\tau-T) Q_F^{-1}\widetilde{F}(\tau-T) \\
= \;& [\widetilde{F}^T(\tau) Q_F^{-1}\widetilde{F}(\tau) - \widetilde{F}^T(\tau) Q_F^{-1}\widetilde{F}(\tau-T)] \\
& +[\widetilde{F}^T(\tau) Q_F^{-1}\widetilde{F}(\tau-T) - \widetilde{F}^T(\tau-T) Q_F^{-1}\widetilde{F}(\tau-T)] \\
= \;& -\widetilde{F}^T(\tau) Q_F^{-1}\delta F(\tau) - \widetilde{F}^T(\tau-T) Q_F^{-1}\delta F(\tau) \\
= \;& -(2\widetilde{F}^T(\tau) + \delta F(\tau)) Q_F^{-1}\delta F(\tau) \\
\leqslant \;& -2\widetilde{F}^T(\tau) Q_F^{-1}\delta F(\tau) \\
= \;& -2\widetilde{F}^T(\tau)[\varepsilon(\tau) - \beta(\tau)F(\tau) + Q_r^T\delta\xi_r(\tau)] \qquad (29)
\end{aligned}$$

Then, similarly, we have

$$\begin{aligned}
& \text{tr}[\widetilde{K}_S^T(\tau) Q_S^{-1}\widetilde{K}_S(\tau) - \widetilde{K}_S^T(\tau)(\tau-T) Q_S^{-1}\widetilde{K}_S(\tau-T)] \\
& \leqslant -2\text{tr}\{\widetilde{K}_S^T(\tau)[\varepsilon(\tau)x^T(\tau) - \beta(\tau)K_S(\tau)]\} \\
& \text{tr}[\widetilde{K}_D^T(\tau) Q_d^{-1}\widetilde{K}_D(\tau) - \widetilde{K}_D^T(\tau-T) Q_D^{-1}\widetilde{K}_D(\tau-T)] \\
& \leqslant -2\text{tr}[\widetilde{K}_D^T(\tau)(\varepsilon(\tau)\dot{x}^T(\tau) - \beta(\tau)K_D(\tau))] \qquad (30)
\end{aligned}$$

Substituting Ineqs. (29) and (30) into Eq.(28) and considering Ineq. (27) yields

$$\begin{aligned}
\delta J_r + \delta J_c \leqslant & \int_{t-T}^{t} -\delta\xi_r^T L\delta\xi_r - \widetilde{F}^T(\varepsilon - \beta F) \qquad (31)\\
& -\text{tr}[\widetilde{K}_S^T(\varepsilon x^T - \beta K_S)] - \text{tr}[\widetilde{K}_D^T(\varepsilon\dot{x}^T - \beta K_D)]\, d\tau\,.
\end{aligned}$$

The rest is to deal with the residual in the above inequality, which is similar to that in [2]. For completeness, we show the outline in the following. In particular, we consider the time derivative of J_e

$$\begin{aligned}
\dot{J}_e &= \varepsilon^T M(q,\dot{q})\dot{\varepsilon} + \frac{1}{2}\varepsilon^T \dot{M}(q,\dot{q})\varepsilon \\
&= \varepsilon^T M(q,\dot{q})\dot{\varepsilon} + \frac{1}{2}\varepsilon^T C(q)\varepsilon \qquad (32)
\end{aligned}$$

as [5]

$$z^T\dot{M}z \equiv z^T C z \quad \forall z\,. \qquad (33)$$

Considering the closed-loop dynamics Eq.(11), above equation can be written as

$$\dot{J}_e(t) \equiv \varepsilon^T(\widetilde{F}^T + \widetilde{K}_S^T\, x + \widetilde{K}_D^T\, \dot{x} - \Gamma\varepsilon)\,. \qquad (34)$$

Integrating $\dot{J}_e$ from $t-T$ to t and considering Ineq. (31), we obtain

$$\begin{aligned}
\delta J &= \delta J_c + \delta J_r + \delta J_e \\
&\leqslant \int_{t-T}^{t} -\varepsilon^T\Gamma\varepsilon - \delta\xi_r^T L\delta\xi_r \\
&\quad +\beta[\widetilde{F}^T F + \text{tr}(\widetilde{K}_S^T K_S + \widetilde{K}_D^T K_D)]\, d\tau \\
&= \int_{t-T}^{t} -\varepsilon^T\Gamma\varepsilon - \delta\xi_r^T L\delta\xi_r \\
&\quad -\beta[\widetilde{F}^T\widetilde{F} + \text{tr}(\widetilde{K}_S^T\widetilde{K}_S + \widetilde{K}_D^T\widetilde{K}_D)] \\
&\quad +\beta[\widetilde{F}^T F^* + \text{tr}(\widetilde{K}_S^T K_S^* + \widetilde{K}_D^T K_D^*)]\, d\tau\,. \qquad (35)
\end{aligned}$$

A sufficient condition for $\delta J \leqslant 0$ is

$$\begin{aligned}
& \lambda_\Gamma\|\varepsilon\|^2 + \lambda_L\|\delta\xi_r\|^2 + \beta(\|\widetilde{F}\|^2 + \|\widetilde{K}_S\|^2 + \|\widetilde{K}_D\|^2) \\
& -\beta(\|\widetilde{F}\|\|F^*\| + \|\widetilde{K}_S\|\|K_S^*\| + \|\widetilde{K}_D\|\|K_D^*\|) \geq 0 \qquad (36)
\end{aligned}$$

where λ_Γ and λ_L are the minimal eigenvalues of Γ and L, respectively. Therefore, $\|\varepsilon\|$, $\|\delta\xi_r\|$, $\|\widetilde{F}\|$, $\|\widetilde{K}_S\|$ and $\|\widetilde{K}_D\|$ are bounded. In particular, they satisfy

$$\begin{aligned}
& \lambda_\Gamma\|\varepsilon\|^2 + \lambda_L\|\delta\xi_r\|^2 + \frac{\beta}{2}(\|\widetilde{F}\|^2 + \|\widetilde{K}_S\|^2 + \|\widetilde{K}_D\|^2) \\
& \leq \frac{\beta}{2}(\|F^*\|^2 + \|K_S^*\|^2 + \|K_D^*\|^2)\,. \qquad (37)
\end{aligned}$$

By choosing large λ_Γ and λ_L, $\|\varepsilon\|$ and $\|\delta\xi_r\|$ can be made small.

B. Proof for minimisation of overall cost when neglecting damping

Consider the cost function

$$J_r' \equiv \frac{1}{2}\int_{t-T}^{t} (x_r - x_d)^T K_S^{*T} Q_r^T (x_r - x_d)\, d\tau\,. \qquad (38)$$

Following similar procedures to Ineqs. (26), (27), we obtain

$$\delta J_r' \leqslant \int_{t-T}^{t} [-L^T \delta x_r + Q_r(\widetilde{F} + \widetilde{K}_S x_r)]^T \delta x_r \, d\tau \tag{39}$$

Considering further the cost function

$$J_c' \equiv \frac{1}{2} \int_{t-T}^{t} \widetilde{F}^T Q_F^{-1} \widetilde{F} + \mathrm{vec}^T(\widetilde{K}_S) Q_S^{-1} \mathrm{vec}(\widetilde{K}_S) \, d\tau \, . \tag{40}$$

and following similar procedures from Ineqs.(28) to (31), we obtain

$$\begin{aligned} &\delta J_r' + \delta J_c' \qquad (41) \\ &\leqslant \int_{t-T}^{t} -\delta x_r^T L \delta x_r - \widetilde{F}^T(\varepsilon - \beta F) - \mathrm{tr}[\widetilde{K}_S^T(\varepsilon\, x^T - \beta K_S)] \, d\tau \, . \end{aligned}$$

The rest is similar to the case with damping and thus omitted.

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